3.2: Understanding Relations and Functions

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Explore Understanding Relations

A relation is a set of ordered pairs (x, y) where x is the input value and y is the output value. The domain is all possible inputs of a relation, and the range is all possible outputs of a relation. For example, the given relation represents the number of whole-wheat cracker boxes sold and the money earned.

{(1, 4), (2, 8), (3, 12), (4, 16)}.

Domain: {1, 2, 3, 4} Range: {4, 8, 12, 16}

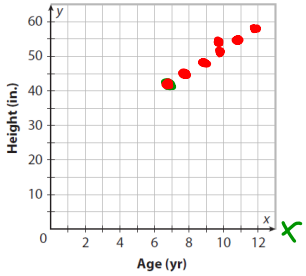
(A) For the following relation, the input, x, is the ages of boys and the output, y, is their corresponding height, in inches.

{(7, 41), (8, 45), (9, 49), (10, 52), (10, 53), (11, 55), (12, 59)}

(B) Fill the values in the table.

x	y
7	41
8	45
9	49
10	52
10	53
11	55
12	59

(C) Plot the points on the graph.



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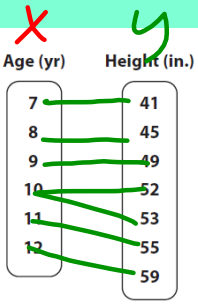
(D) Complete the mapping diagram.



(E) State the domain of the relation.

{7, 8, 9, 10, 10, 11, 12}

(F) State the range of the relation.

{41, 45, 49, 52, 53, 55, 59}

Reflect

1. **Discussion** The number 10 appears twice in the x column of the table. How many times is it written in the domain? Explain.

Twice. There are 2 different points with a domain of 10.

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Explain 1 Recognizing Functions

A function is a type of relation in which there is only one output value for each input value.

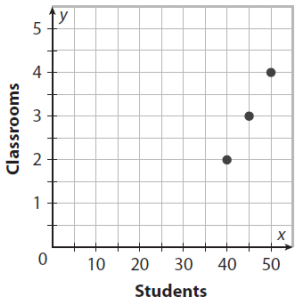
For every input value, there is a unique output value.

Example: $y = x^2$. When $x = 3$, y will always be equal to 9.

Example 1 **Give the domain and range of each relation. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.**

(A) The given relation represents the number of students and the number of classrooms the school has to have for the corresponding number of students.

Students x	Classrooms y
40	2
45	3
50	4



Domain: {40, 45 50}

The domain represents the number of students.

Range: {2, 3, 4}

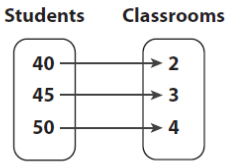
The range represents the number of classrooms.

For an input of 40 students, there is an output of 2 classrooms.

For an input of 45 students, there is an output of 3 classrooms.

For an input of 50 students, there is an output of 4 classrooms.

This relation is a function. Each domain value is paired with exactly one range value.



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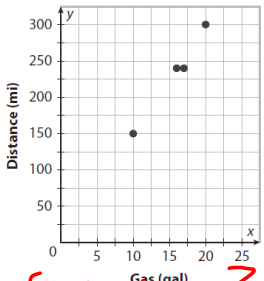
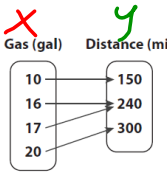
(B) The given relation represents the amount of gas in gallons and the distance traveled in miles from that amount of gas.

Gas (gal)	Distance (mi)
10	150
16	240
17	240
20	300





Domain: {10, 16, 17, 20} gas (gallons)

The domain represents

Range: {150, 240, 240, 300}

The range represents Distance (miles)

For an input of 10 gallons, there is an output of 150 miles.

For an input of 16 gal., there is an output of 240 miles.

For an input of 17 gal, there is an output of 240 miles.

For an input of 20 gal., there is an output of 300 miles.

This relation is a function. Each domain value is paired with one range value.

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Reflect

2. If each month in a year was paired with all the possible numbers of days in the month, will the result be a function? Explain.

months days
x y
Yes. Months do not repeat in one year.

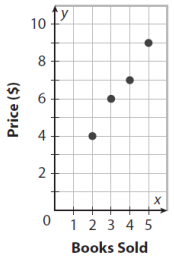
Your Turn

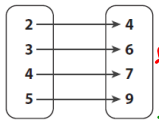
Give the domain and range of each relation and interpret them in context. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.

3. The relation represents the number of books sold and the price for the corresponding number of books.

x y

Number of books sold	Price ($)
2	4
3	6
4	7
5	9





Domain: {2, 3, 4, 5}
Range: {4, 6, 7, 9}

For an input of 2 bks sold, there's an output of $4.
For an input of 3 bks, there's an output of $6.
For an input of 4 bks, there's an output of $7.
For an input of 5 bks, there's an output of $9.

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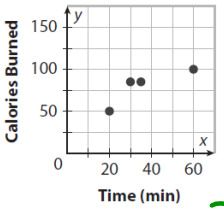
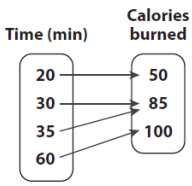
4. The relation represents the time spent exercising and the number of calories burned during that time.

x y

Time (min)	Calories burned
20	50
30	85
35	85
60	100





Domain: {20, 30, 35, 60}
Range: {50, 85, 85, 100}

Yes it is a function.
The domain does not repeat.

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Explain 2 Understanding the Vertical Line Test

A test, called the *vertical line test*, can be used to determine if a relation is a function. The **vertical line test** states that a relation is a function if and only if a vertical line does not pass through more than one point on the graph of the relation.

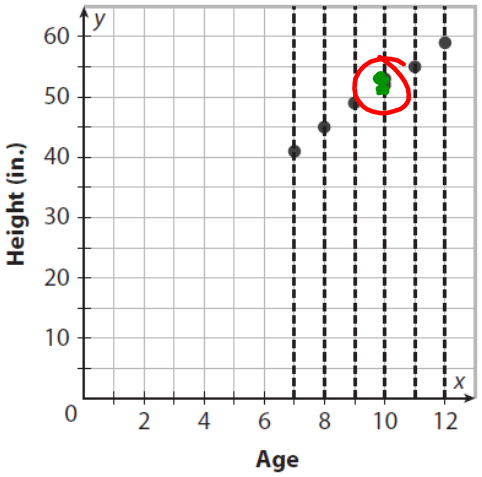
Example 2 **Use the vertical line test to determine if each relation is a function. Explain.**

(A) Draw a vertical line through each point of the graph.

Does any vertical line touch more than one point? Yes

Since a vertical line does pass through more than one point, the graph fails the vertical line test. So, the relation is not a function.

not a function. The x-values repeat @ 10.



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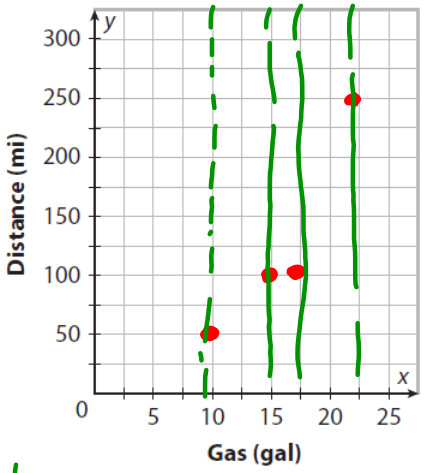
(B) Draw a vertical line through each point of the graph.

Does any vertical line touch more than one point? no

Since a vertical line does not pass through more than one point, the graph passes the vertical line test. So, the relation is a function.



Reflect

5. Why does the vertical line test work?

shows you if the domain repeats.

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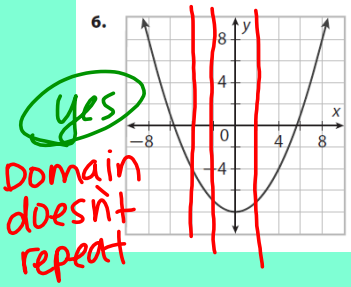
Your Turn

Use the vertical line test to determine if each relation is a function.

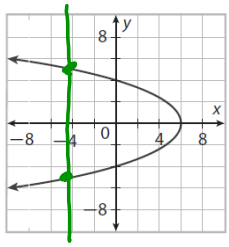
6.



yes

Domain doesn't repeat

7.



NO

x-values repeat.

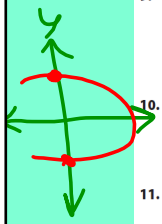
Elaborate

8. How can you use a mapping diagram to determine the domain and the range of a relation?

Draw an arrow from each x-value to it's y-value

9. **Discussion** For a discrete function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

NO. If the number of range values is greater than the number of domain values, then there must be a domain value that is paired with more than one range value.



10. Is a relation a function if its graph intersects the y-axis twice?

NO. The x-values repeat at 0.

11. **Essential Question Check-In** You are asked to determine if the relation $y = x^2 - 8x + 4$ is a function. What would be the best way to represent this relation in order to determine if it is a function or not? Explain.

graph in calculator & use vertical line test.

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HOMEWORK TIME!

Pages 114-118
*OMIT #1 & 24

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Evaluate: Homework and Practice

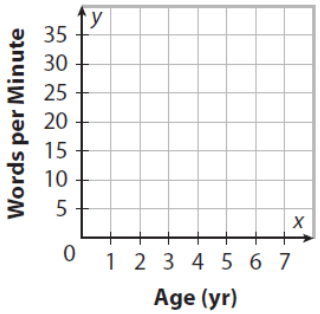
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Express each relation as a table, as a graph, and as a mapping diagram.

1. The relation represents ages of students and the number of words they can write per minute.

$\{(5, 10), (6, 20), (6, 23), (7, 35)\}$

x	y



Age (yr) | Words per Minute

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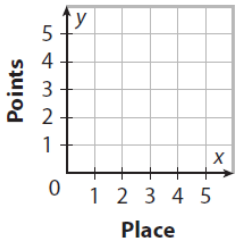
Express each relation as a table, as a graph, and as a mapping diagram.

2. The relation represents the place won in a track meet and the number of points that place finish is worth.

$\{(1, 5), (2, 3), (3, 2), (4, 1), (5, 0)\}$

x	y



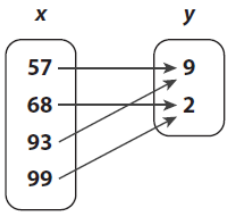
Place | Points

State the domain and range of each relation.

3.

x	y
2	5
7	8
8	15
11	12
15	19

4.



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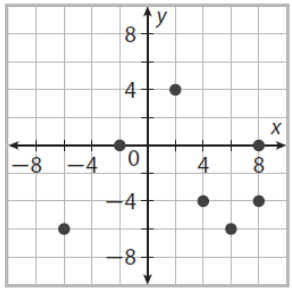

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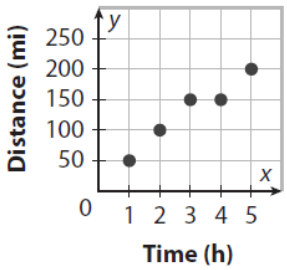
State the domain and range of each relation, interpret in context, and explain if it is a function or not.

6. The relation represents the age of each student and the number of pets the student has.

Age	Number of Pets
6	3
8	2
9	0
11	1
11	2

7. The relation represents time driven in hours and the number of miles traveled at the end of each hour.



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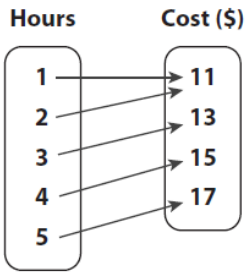
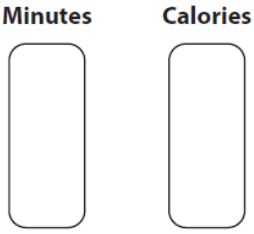
State the domain and range of each relation, interpret in context, and explain if it is a function or not.

8. The relation represents the number of hours a person is able to rent a canoe and the cost of renting the canoe for that many hours.



9. A person can burn about 6 calories per minute bicycling. Let x represent the number of minutes bicycled, and let y represent the number of calories burned. Create a mapping diagram to show the number of calories burned by bicycling for 60, 120, 180, or 240 minutes.



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10. The table represents a sample of ages of people and their shoe size.

Age	Shoe Size
x	*y*
16	7
16	8
19	10
22	10
25	10.5
28	11

11. An electrician charges a base fee of $75 plus $50 for each hour of work. The minimum the electrician charges is $175. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work.

x	*y*

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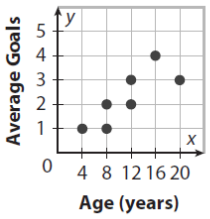
12. The graph represents the average soccer goals scored for players of different ages. Determine the domain and range of the relation in context and explain whether or not this represents a function.



Express each relation as a mapping diagram and explain whether or not the relation represents a function.

13. $\{(13, 33), (17, 25), (22, 22), (25, 17), (33, 17)\}$

14. $\{(1, 2), (5, 2), (5, 4), (7, 6), (11, 6) (11, 8)\}$

x	*y*

x	*y*

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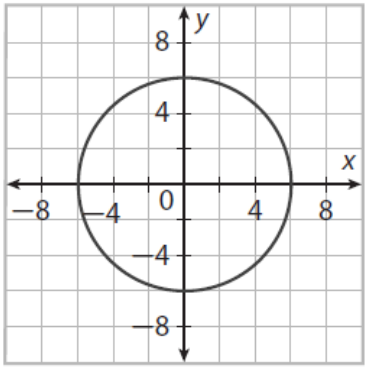
Use the vertical line test to determine if each relation is a function.

15.



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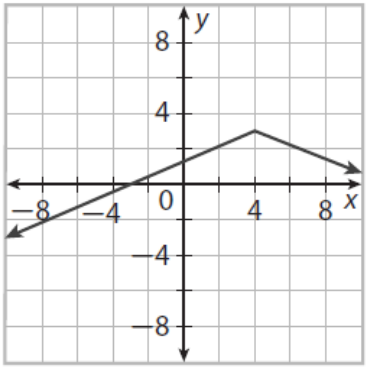

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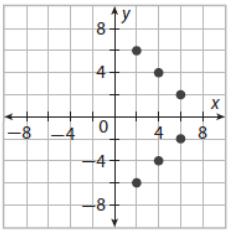
Use the vertical line test to determine if each relation is a function.

17.



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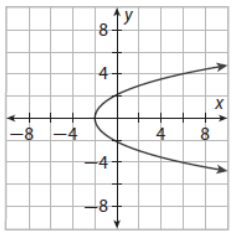

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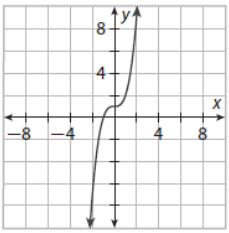

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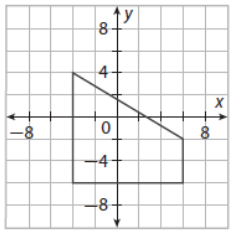

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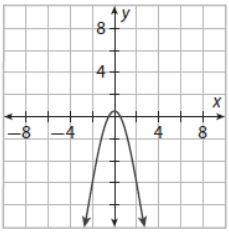

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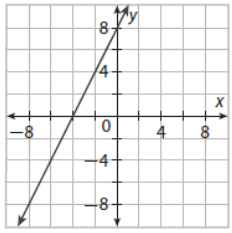

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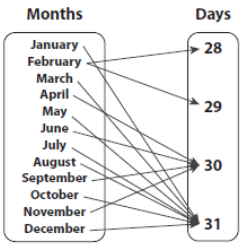
23. Draw Conclusions Examine the mapping diagram. The first set is the months of the year, and the second set is the possible number of days per month. Is the relation a function? Explain.

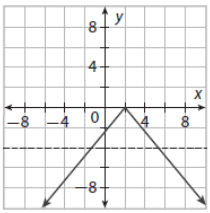


24. Justify Reasoning Tell whether each situation represents a function. Explain your reasoning. If the situation represents a function, give the domain and range.

a. Each U.S. coin is mapped to its monetary value.

b. A $1, $5, $10, $20, $50, or $100 bill is mapped to all the sets of coins that are the same as the total value of the bill.

25. Explain the Error A student was given a graph and asked to use the vertical line test to determine if the relation was a function or not. The student said that the relation failed the vertical line test and the graph was not a function. What error did the student make? Explain the error and give the correct answer.



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